

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Subject Code :

2 0

Test Booklet No. : 00920

TEST BOOKLET
MATHEMATICS

Time Allowed : 2 (Two) Hours

Full Marks : 200

INSTRUCTIONS

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Subject Code shall be written legibly and correctly in the space provided on the Answer Sheet with black ball pen.
2. Space provided for Series in the Answer Sheet is not applicable for Optional Subject. So the space shall be left blank.
3. All questions carry equal marks. Your total marks will depend only on the number of correct responses marked by you in the Answer Sheet.
4. No candidate shall be admitted to the Examination Hall/Room 20 minutes after commencement of distribution of the paper. The Supervisor of the Examination Hall/Room will be the time-keeper and his/her decision in this regard is final.
5. No candidate shall leave the Examination Hall/Room without prior permission of the Supervisor/Invigilator. No candidate shall be permitted to hand over his/her Answer Sheet and leave the Examination Hall/Room before expiry of the full time allotted for each paper.
6. No Mobile Phone, Pager, etc., are allowed to be carried inside the Examination Hall/Room by the candidates. Any Mobile Phone, Pager, etc., found in possession of the candidate inside the Examination Hall/Room, even if on off mode, shall be liable for confiscation.
7. No candidate shall have in his/her possession inside the Examination Hall/Room any book, notebook or loose paper, except his/her Admission Certificate and other connected paper permitted by the Commission.
8. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
9. After you have completed filling in all your responses on the Answer Sheet and the Examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
10. Violation of any of the above Rules will render the candidate liable to expulsion from the Examination Hall/Room and disqualification from the Examination, and according to the nature and gravity of his/her offence, he/she may be debarred from future Examinations and Interviews conducted by the Commission for appointment to Government Service.
11. Smoking inside the Examination Hall/Room is strictly prohibited.
12. This Test Booklet contains three pages for Rough Work at the end.

SEAL

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[No. of Questions : 100]

1. For any two sets A and B , $A \cap (A \cup B)'$ is equal to

- (A) A (B) B
(C) ϕ (D) $A \cap B$

2. If A and B be two non-empty sets, then which one of the following is the result of $(A \cup B) - A$?

- (A) $A - B$ (B) $B - A$
(C) B (D) A'

3. Consider the function $f: W \rightarrow W$ defined by

$$f(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$$

where W is the set of whole numbers. Then the function f is

- (A) one-one
(B) onto
(C) bijective
(D) None of the above

4. If A and B are two sets such that $n(A) = 2$, $n(B) = 3$ then $n(P(A \times B))$ is equal to

- (A) 64 (B) 6
(C) 5 (D) 12

5. With respect to composite composition, the set A_n of all even permutations of degree n , defined on a set S forms a finite group of order

- (A) n (B) $\angle n$
(C) $n/2$ (D) $\angle n/2$

6. If in a group of 65 people, 40 like cricket, 10 like both cricket and tennis, then the number of people who like tennis only and not cricket is

- (A) 15 (B) 25
(C) 35 (D) 45

7. If in a group G , $a^5 = e$, the identity element of G and $aba^{-1} = b^2$ for $a, b \in G$ then $O(b)$ is

- (A) 31 (B) 29
(C) 23 (D) 19

8. The set Q of rational numbers is not a group under the operation $*$ defined by $a * b = \frac{ab}{2}$ for

- (A) $*$ is not a binary operation in Q
(B) $*$ is not associative in Q
(C) there does not exist identity element in Q
(D) some elements of Q have no inverses

9. Let H and K be two subgroups of a group G . Then $H \cup K$ is a subgroup of G if and only if
- (A) either $H \subseteq K$ or $K \subseteq H$
 (B) $H \cap K = \phi$
 (C) $H \subseteq K$ and $K \subseteq H$
 (D) None of the above
10. Let $(Q, +, \cdot)$ be a ring of rational numbers. Then the set of integers Z is
- (A) a subring but not an ideal of Q
 (B) neither a subring nor an ideal of Q
 (C) a subring as well as an ideal of Q
 (D) not a subring but an ideal of Q
11. The inverse of a symmetric matrix is
- (A) symmetric
 (B) skew-symmetric
 (C) diagonal matrix
 (D) None of the above
12. The rank of a $m \times n$ matrix whose every element is unity is
- (A) m (B) n
 (C) mn (D) 1
13. The system of linear equations $AX = B$ is consistent if and only if the coefficient matrix A and the augmented matrix $[AB]$ are of
- (A) same rank
 (B) different ranks
 (C) same number of rows
 (D) None of the above
14. If $x, y, z \in R^+$ and $x^3 + y^3 + z^3 = 81$ then the maximum value of $x + y + z$ is
- (A) $3\sqrt{3}$ (B) 3
 (C) 27 (D) 9
15. The condition in order that the equation $x^3 - px^2 + qx - r = 0$ may have a pair of equal roots is
- (A) $pq = r$
 (B) $pr = q$
 (C) $qr = p$
 (D) None of the above
16. The infinite series
- $$\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \dots \infty$$
- is
- (A) convergent
 (B) oscillatory
 (C) divergent
 (D) None of the above

17. The series

$$\frac{1}{2} + \frac{4}{9}x + \frac{9}{28}x^2 + \dots + \frac{n^2}{n^3 + 1}x^{n-1} + \dots$$

is convergent if

- (A) $x < 1$
- (B) $x > 1$
- (C) $x = 1$
- (D) None of the above

18. Choose the incorrect statement :

- (A) A convergent sequence determines its limit uniquely.
- (B) Every convergent series is bounded.
- (C) A monotonic increasing sequence diverges to $+\infty$ if it is not bounded above.
- (D) A monotonic decreasing sequence is not convergent.

19. $\lim_{x \rightarrow \infty} (\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x})$ is equal to

- (A) 0
- (B) $\frac{1}{2}$
- (C) $\log 2$
- (D) e^4

20. The function f defined by

$$f(x) = \begin{cases} \frac{3|x| + 4 \tan x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

is continuous at $x = 0$ for

- (A) $k = 0$
- (B) $k = 1$
- (C) $k = 7$
- (D) no value of k

21. If $y = \log(\sin x)$, then y_3 is equal to

(A) $\frac{\sin x + \cos^2 x}{\sin^4 x}$

(B) $\frac{2 \cos x}{\sin^3 x}$

(C) $\frac{1 + \cos^2 x}{\sin^3 x}$

(D) $\frac{\sin^2 x + 2 \cos x}{\sin^3 x}$

22. Suppose the function f satisfies the conditions

(i) $f(x + y) = f(x)f(y) \quad \forall x \text{ and } y$

(ii) $f(x) = 1 + xg(x)$ where

$$\lim_{x \rightarrow 0} g(x) = 1$$

Then $f'(x)$ is equal to

- (A) $f(x)$
- (B) $f(x) \cdot g(0)$
- (C) $2f(x) \cdot g(0)$
- (D) None of the above

23. If $\phi(x, y) = x^3 y \sin^{-1}\left(\frac{y}{x}\right)$ and

$$x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} = n\phi$$

then n is equal to

- (A) 4
- (B) 5
- (C) 6
- (D) 7

